Abstract Title: Using Research-based Curriculum to Support Shifts in Teachers' Key Pedagogical Understandings

MSP Project Name: Project Pathways: Opening Routes to Math & Science Success for All Students

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120 word summary:

Research findings from the first four years of a project (*Project Pathways*) focused on improving secondary mathematics and science instruction revealed that teachers' attempts to promote conceptual learning were not supported by their curriculum. In response, the project leaders designed a research-based, conceptually oriented precalculus curriculum for use in the secondary schools. This session reports on the results of one teacher's (Claudia) implementation of these materials. Overall, the curriculum and teacher supports associated with the curriculum resulted in improvements in Claudia's mathematical content knowledge and her pedagogical actions. These improvements helped Claudia support her students' learning processes and problem solving abilities by asking questions that revealed and built on her students' thinking.

Section 1: **Questions for dialogue at the MSP LNC**.

What shifts occur in a precalculus teacher's mathematical content knowledge and pedagogical actions when a teacher uses research-based and conceptually oriented precalculus curriculum? How do these shifts support student learning?

Section 2: **Conceptual framework**.

Context of Work

Project Pathways is an ongoing initiative that focuses on the professional development of secondary mathematics and science teachers. As a project in its sixth year, *Project Pathways* interventions have included graduate courses, workshops, and school-based Professional Learning Communities (PLCs) facilitated by trained peers and/or Pathways faculty. The project design intends to support the mathematics and science teachers in achieving inquiry-oriented classrooms that promote conceptual learning. Additionally, *Project Pathways* strives to generate communities of teachers that work together to reflect upon and improve their teaching and, in turn, improve their students' success in mathematics. We define *student success* as improved learning of concepts that are foundational for continued mathematics and science course taking, as assessed by PCA (and FCI), improved problem solving behaviors, and improved beliefs about the methods and nature of mathematics.

During the first four years (cohorts 1-4) of *Project Pathways*, research (M. Carlson, Moore, Bowling, & Ortiz, 2007; Clark, Moore, & Carlson, 2008) revealed that the interventions were producing shifts in the teachers' mathematical knowledge and the nature of their mathematical discussions. As an example, the teachers involvement in PLCs, along with facilitator coaching from project members, led to many of the teachers holding each other to higher standards for what constituted *speaking with meaning* (Clark, et al., 2008) when discussing a mathematical topic or providing a justification for solutions and statements. Paralleling this finding, research (M. Carlson, et al., 2007) also revealed shifts in the teachers' attempts to adopt the thinking and understanding of other individuals, which Piaget (1955) defined as *decentering*. As teachers improved their ability and propensity to decenter, their interactions with fellow teachers became more mathematically substantive and productive during their PLC meetings. However, the teachers involved in the first four years of the project only made modest shifts in their classroom practices, where these shifts lagged significantly behind the shifts made in the PLCs.

Observations and interactions with the teachers involved in the first four cohorts revealed that the teachers' curriculum did not support their attempts to achieve a student-centered and conceptually oriented classroom. Complicating the issue, the curriculum the teachers were using in their classroom did not align with the mathematical and scientific ideas they were learning in the *Pathways* graduate courses and PLC meetings. In addition, department chairs frequently exhibited instruction that was procedurally oriented and they did not possess the knowledge, interest or insight to lead other teachers in shifting their instruction. In response, we implemented the *Pathways Professional Development Model* in schools (*Pathways Adopted Schools*) in which the school administrators support the project goals, and all mathematics and science teachers participated in some components of the intervention. Pathways cohorts 5 and 6 include four such *Pathways Adopted Schools*.

The *Pathways Professional Development Model* includes a significant focus on teacher involvement in PLCs that create and implement student-centered and conceptually oriented lessons. During the first year of working with cohorts 5 and 6, improvements were documented in student learning using research-based assessment instruments (e.g., *Precalculus Concept Assessment*, *Mechanics Baseline Inventory*), although the shifts were not as dramatic as we had hoped. The teachers also found revising their curriculum to be a daunting and exhaustive task. In response to these obstacles and shortcomings, the project leadership chose to offer a research-based, conceptually oriented precalculus curriculum (M. P. Carlson & Oehrtman, 2009) for use by one precalculus teacher (Claudia) within a *Pathways School*. These curriculum materials were showing improvements with college level students, and we conjectured that the combination of teacher support tools and student curriculum tasks would support a high school teacher in achieving the desired shifts in her practice and students' learning. We were also interested in understanding the role of the curriculum in supporting shifts in a teacher's mathematical content knowledge for teaching and pedagogical actions in her classroom.

For this study we leverage Silverman (Silverman, 2005) and Silverman and Thompson's (2008) constructs of Key Developmental Understandings (KDU) and Key Pedagogical Understandings (KPU) to examine the mathematical knowledge for teaching precalculus level mathematics. Like Silverman and Thompson (2008), we believe that a teacher's mathematical conceptions and reasoning abilities influence their pedagogical choices and pedagogical actions during teaching. If a teacher's conception of proportional reasoning consists of cross-multiplying, the teacher is likely to focus on teaching students to cross-multiply. On the other hand, if a teacher's conception of proportionality includes an image of two quantities such that if one is scaled by a factor the other must be scaled by the same factor to maintain a constant ratio, the teacher will make pedagogical choices and actions to promote this way of thinking about proportionality. Such a teacher would be said to have a KDU (Silverman & Thompson, 2008; Simon, 2006), which includes a scheme of meanings that connect a spectrum of ideas. Silverman states that a person is developing a KPU when she becomes reflectively aware of a KDU and realizes that her students would benefit from thinking this way. According to Silverman, "A [KPU] is a person's transformation of a [KDU]…to a way of understanding how this KDU could empower students' learning of related ideas were they do have it" (2005, p. 15). Our research intended to gain insights into the role of the precalculus curriculum in Claudia's development of KPU and KDU, and how resulting shifts in Claudia's teaching impacted her students' learning.

Claims

Research-based, conceptually oriented curriculum can support teachers in shifting to a student-centered classroom that promotes students' learning conceptual and well-connected mathematics. Such curriculum supports teachers in achieving shifts in their mathematical content knowledge and pedagogical actions, leading to the development of conceptual KDU and KPU. In turn, the teacher can better support student learning through questioning and problem solving activities that build on students' thinking.

Section 3: **Explanatory framework**.

Evaluation, Research Design, and Data Analysis

During Claudia's implementation of the precalculus curriculum, our research focus focused on her classroom sessions, the PLC meetings she attended, and her interactions with project members. While using the curriculum, Claudia attended a weekly 50-minute PLC composed of 4-6 trigonometry teachers, and she sporadically attended a 3-hour trigonometry PLC with two other trigonometry teachers. The PLCs primarily collaborated in examining what was involved in understanding and learning key ideas of their courses. Three *Project Pathways* members regularly visited her classroom to make observations, answer questions she had about the curriculum, and offer suggestions on how to improve her teaching.

Claudia's classroom was videotaped two class periods a day, and these videos were digitized for analysis. All PLC meetings were also videotaped and digitized, and field notes were taken during Claudia's meetings with Pathways members. All data were used to gain insights into her mathematical knowledge and pedagogical actions. As Claudia interacted with her students during class sessions, her responses and questioning offered insights into her and her students' mathematical understandings and thinking. Similarly, her interactions during the PLC meetings offered glimpses into her content knowledge, particularly when the PLC sessions focused on discussing how students learn various mathematical topics. Relative to Claudia's meetings with Pathways members, she often asked for clarification on the mathematical ideas driving the curriculum. Claudia also discussed questions she had about the solutions and reasoning her students were providing. Collectively, these data were used to characterize Claudia's mathematical content knowledge, her pedagogical actions, and improvements in her students' learning.

In the Pathways Precalculus curriculum, students are introduced to trigonometry beginning with a lesson on angle measure. The subsequent lessons build on the concepts of angle measure and proportionality to introduce trigonometric functions. Specifically, students are given a circular motion context (e.g., a bug on the tip of a fan blade) and asked to determine how the bug's vertical distance above the center of the fan varies with the angle measure swept out by bug. When beginning this problem, Claudia asked the students the following question: Does the angle measure change if the radius of the fan is changed, but the distance the bug travels (measured in radians) is not changed? Excerpt 1 presents the conversation between two students and Claudia in response to this question.

Excerpt 1

Student 1:	How does the angle change if the radius changes? The angle stays the same.
Student 2:	Yeah, but if the distance traveled around stays the same, but the radius changes then it will be a different amount of radians.
Student 1:	Oh yeah.
Student 2:	It will be inversely proportional.
Claudia:	It will be a different amount of radians?
Student 2:	The angle measure.
Student 1:	Well the distance the bug travels.
Student 2:	Because as the radius gets bigger, the arc length will be a lesser portion of the circle meaning a lesser angle.
Claudia:	Ok, so draw, ok so let's take any two circles, if you were to cut out a 90 degree angle, actually, draw a circle inside of a circle for me.
Student 2:	The arc length is…
Claudia:	Now draw another circle - yeah.
Claudia:	Ok, now do you have a Wikki Stix, does anyone, you do?
Claudia:	Ok, first for the inside circle I want you to mark off one radian of the circumference.
Student 2:	Can we just take a Wikki measure, do I have to use radians?
Claudia:	She is trying to find the center of her circle though, is what she is doing. Here let me get you a smaller Wikki Stix. Here is a skinner one it might be easier to use.

Student 1:	It is two and half so it would be like in the middle
Claudia:	Yeah so for the small circle mark off one radian of the circumference. Ok do the same thing for the bigger circle.
Student 2:	One radian?
Claudia:	Yeah one radian
Student 2:	But the radian of an angle measure is not arc measure, the radian and arc measure are the same. Not the angle measure are the same…
Student 1:	One radian is the same…
Claudia:	What is the same about one radian?
Student 1:	The same angle?
Claudia:	Yes, what is the not the same about one radian?
Student 1:	The arc length?
Claudia:	Yes, do you see that Student 2?
Student 1:	But then what is distance traveled? Is that the arc length?
Student 2:	Yes.

During *Excerpt 1*, Claudia enters the conversation to receive some clarification on what the students have claimed (see lines 1-6). After gaining additional information about how the students are thinking (see lines 7-10), Claudia proceeds by having the students return to thinking about radian angle measure as measuring along arcs on circles of various sizes (see lines 11-29). As a result, the students noted that one radian on different circles corresponded to the same angle, but that the magnitude of the arc length on each circle was different (see lines 28-34). This interaction demonstrates Claudia making a pedagogical move to have her students reflect back on measuring an arc length and it's relationship to angle measure. Also, Claudia presented this question immediately previous to the students exploring the sine and cosine functions, which suggests that Claudia found this to be an important way of reasoning for that task. These actions imply that Claudia had developed a KPU of angle measure and trigonometric functions that consisted of thinking about angle measure in terms of an arc. As a result, Claudia questioned her students in ways that enabled them to make this connection.

Claudia's development of this KPU was further verified during her interactions with her trigonometry PLC, which occurred after she had taught the trigonometry lessons. During the year previous to Claudia's use of the curriculum, it had not occurred to the PLC members that developing an arc length image of angle measure could be leveraged to promote students constructing meaningful understandings of the sine and cosine functions. However, after using the curriculum, Claudia emphasized this KPU when her PLC members were designing a trigonometry lesson. Initially, her fellow teachers chose to design a lesson focused on applications of trigonometric functions. However, Claudia objected to this choice because she was unable to imagine teaching applications of trigonometric functions conceptually without teaching angle measure in a way that supported a conceptual approach to trigonometric functions. This action by Claudia supports that she developed a KPU of trigonometric functions and angle measure that consisted of an arc length approach to angle measure. Also, it appears that Claudia's use of the Pathways precalculus curriculum promoted her development of this KPU, as she did not articulate such a scaffolding of ideas during the previous years' PLC meetings.

Key Insights

Analysis of Claudia's classroom revealed significant shifts in her teaching practices that reflected improvements in KDU for teaching and her ability to make pedagogical actions based on these KDU (e.g., she developed KPU). These findings lead us to conclude:

- The choice of curriculum is critical in supporting teachers in improving their practice.
- Research-based, conceptually oriented curriculum can influence teachers' KDU and KPU, which can lead to improvements in their classroom practice and students' learning.
- Improved KDU and KPU can support a teacher in questioning her/his students in ways that build on their students' thinking and lead to their students constructing conceptual understandings.

- KDU and KPU are critical components of a teachers' practice, and curriculum must support teachers in developing these ways of understandings such that they support student learning.

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