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A Faculty Learning Community Implements Research-based Curriculum and Pedagogy to Redesign Precalculus

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ABSTRACT
This article reports on a reconceptualization of the Precalculus course experience at Florida International University. We discuss the details of the redesign process – unified within a faculty learning community (FLC) model – along with a broader change in the course sequence leading up to Calculus. We provide data, including the comparison between the treatment and control groups.

KEYWORDS
Faculty learning community; precalculus; course redesign; constructivism

1. INTRODUCTION

In 2018, we began the process of redesigning our precalculus course to support the adoption of a research-based curriculum. We found that by creating a faculty learning community that focuses on advancing our collective understanding of what is involved in understanding and learning precalculus ideas, you can both increase the quality of learning and shift faculty instructional practices to be more meaningful and engaging for students. These practices concentrate on implementing dynamic, mental preparatory assignments and fostering student covariational thinking, both of which enable students to become architects of their own knowledge.

Efforts to implement and scale innovative curricula must address inherent challenges. This paper details the process that we engaged in to implement this research-based curriculum and pedagogy and address these challenges. The steps taken were wide-ranging, going from gaining administrative support to instructional faculty working together on designing novel assessment questions. We discuss results from the first year of the project and offer suggestions for other faculty learning groups interested in implementing research-based curricula.

2. BACKGROUND

Changing demographics, the need for four-year degrees, and the emerging central role of STEM in the modern economy are drivers of strategic priorities, growth, and change at Florida International University (FIU). FIU is a large, urban, public research university with over 80% of its undergraduates from underrepresented

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minority groups [15]. In order to better serve its student population and the broader community, the university has advocated for transformative change in higher education for over a decade. With more than 14,000 STEM majors and roughly 8000 students passing through its lower-division math courses every year, FIU has focused on realizing broad, high-impact solutions through evidence-based practices.

Historically, the transition from College Algebra to Calculus 1 involved only one precalculus course. In Fall 2014, FIU moved to a longer preparatory math sequence based on students' incoming placement scores. This change added a precalculus algebra course that, along with a trigonometry course, constituted a six-credit, two-course sequence designed to prepare students for Calculus I. This revised sequence to Calculus I, while increasing content coverage, was long and exacting, and its intended effect of bridging to Calculus had a low reach: of the STEM degree-seeking FIU students attempting any of the courses below Calculus 1 during the period from Fall 2014 to Spring 2016 only 52.3% ended up attempting Calculus 1 at FIU during the period from Spring 2015 to Fall 2018. Within the university, a desire to impact retention rates, decrease total student credit hours, and increase both 6-year and 4-year graduation rates because of a new State University System of Florida (SUS) performance-based funding model [26] turned the focus of most curricular reform work to innovations that would generate greater student success.

In response, we created a small faculty learning community to identify a combined strategy of prioritizing the use of the one-course precalculus format, together with incorporating a research-based, STEM-oriented curriculum to prepare students for problem-solving in Calculus. The one-course precalculus format had utilized a traditional set of instructional strategies, so this presented an important opportunity to rework the teaching and learning experience. FIU's capacity for the successful adoption of a research-based curriculum in a STEM discipline had already been established by the FIU Physics Education Research Group, founded in 2003, which adapted and implemented Modeling Instruction [3, 4] and utilized a Learning Assistant Program [18] to support students. Additional strands of success were evident in Biology's adoption of Peer-Led Team Learning [1] and General Chemistry's implementation of POGIL, also using learning assistants [27].

Reform efforts that combined active-learning, group work, learning assistants, and faculty collaboration had been successfully integrated into other lower-division mathematics courses at FIU. A Mastery Math Lab resulted from a United States Department of Education Title V grant and an internal FIU technology award [25]. These proven practices – referred to within the university as the Mastery Math Program – created a framework that was natural for the implementation of a research-based STEM-oriented curriculum that would provide a richer precalculus experience that engages students in understanding and applying key mathematical ideas and processes, such as amounts of change, quantitative reasoning, functions categorized by their rates of change, and dynamic mathematical objects.

3. FACULTY LEARNING COMMUNITIES

Supporting teacher development and building campus-wide partnerships are important ingredients in improving instruction. Faculty learning community (FLC) programs [12] have helped to attain these goals. To this end, a topic-based FLC model was established around the Precalculus redesign effort, including qualities and components found in Cox's model ([12, Appendices A and B]). In particular, we set our paper in relation to the following four central conditions of Furco and Moely, distilled from the research literature, for the purpose of advancing faculty buy-in and support [17, pg. 129]:

(1) Faculty have an explicit understanding of the goals of the innovation.
(2) There exist practical opportunities for teacher learning.
(3) Faculty identify a long-range commitment to administrative support.
(4) There is alignment with institutional rewarding of teaching excellence.

After carefully researching alternative curricula, both standards-based and traditional, for this reform effort, we chose the *Precalculus: Pathways to Calculus* curriculum, an inquiry-based curriculum based on decades of research knowledge on student learning [10]. The *Pathways* curriculum is described to students as a way to "see a purpose for learning and understanding the ideas of precalculus, while also helping you acquire critical knowledge and ways of thinking" ([10, pg. iv], *Reprinted with permission by Marilyn Carlson (Copyright 2020)*), incorporating a scaffolded approach to building on students' prior knowledge while simultaneously helping them reimagine several core math essentials within a more conceptual framework. The research that informed the curriculum design and refinement calls for students to be supported in developing confidence and competence in their ability to make sense of quantitative relationships in applied problems and to strengthen their conceptions of key ideas such as rate of change and function [5, 7, 8, 19, 20, 22]. These are essential to the processes involved in calculus-related rates and optimization problems. This increased focus on developing students understanding of ideas, the ability to construct and interpret graphs, and writing meaningful function formulas, required that we shift our exams away from a predominant procedural focus.

While later iterations of the FLC included nearly a dozen members, the goal of being able to collegially synthesize different points of view during the principal design phase was best accomplished with a smaller group. The initial FLC contained four math faculty, one of whom had first done a pilot of the curriculum in College Algebra, where data on student learning was gathered. The other three members all had considerable roles in prior lower-division redesign initiatives that spanned a breadth of courses. Completing the team was an affiliate faculty administrator from our center for teaching and learning. Monetary stipends were provided to all faculty for the initial work, and there was institutional commitment to support advancing the scholarly teaching dimensions of the project. Work was done in quiet on-campus areas outside of the math department building, which facilitated a low-stakes and enjoyable working environment.

The FLC met weekly throughout the summer to create our local version of the *Pathways* course that fit into the Mastery Math Program framework used by other introductory math courses at FIU. This meant that the class had to have both online (pre-class assignments, weekly homework) and written (in-class group work, written homework) components, and a weekly requirement for students to spend time working in the Mastery Math Lab, that was staffed with learning assistants in the classroom. The meetings were also used to clarify our student learning goals, identify key conceptual understandings and procedural skills, and consider the flow of ideas within a module and how to make that flow explicit to students. Between meetings, individual members reviewed the existing curricular materials to find and submit problems that were congruent with the course learning goals and were appropriate for small group work. We selected problems from the online *Pathways* platform and the *Pathways* workbook that we hypothesized would achieve these learning goals. We concurrently developed and worked with *Pathways* authors to supplement *Pathways* with additional content that was required for our course but not yet available in the *Pathways* stand-alone course.

The group sustained consensus decision-making on the selection of pre-class and in-class questions as well as determining a balance between procedural and conceptual content. For example, when teaching rational functions and limits, we wanted to ensure that students walked away with the reasoning behind asymptotic behavior, in addition to producing a rough sketch of the graph of a rational function. The work was reviewed again with the goal of making the workload more manageable for students. The development process included designing a rubric for grading assessment questions that would more easily promote inter-rater reliability together with establishing a system to produce more learning-centered feedback for students. Concurrently, the FLC faculty produced open-ended assessment items aimed at revealing student thinking.

4. STEPS TO IMPLEMENTATION

Once our FLC team had selected the curriculum, we met with key departmental leaders and stakeholders prior to finalizing the course content for FIU students. We provide an overview of this process in a chronological listing that follows. We also acknowledge that other universities, dependent on their local environment and institutional constraints, will likely have other priorities for launching a research-based course.

Step 1: Garner support from college and departmental administration.

The initial task was to present the vision to the Math Task Force, an FIU ad hoc committee put in place, starting Fall 2017, for the purpose of reviewing lower-division math programs. The task force included deans from STEM disciplines, leadership from student success offices and the Center for the Advancement of Teaching, alongside the Chairperson and other faculty from the Department of Mathematics and Statistics. The presentation was in-person, describing the twofold backward design of the curriculum: to support problem-solving processes essential in STEM and the growth of mathematical ideas needed for Calculus.

The committee embraced our creation of a more meaningful precalculus experience that not only had an empirical basis but allowed more students to complete the content in one semester, thereby shortening the math pathway to their upper-division STEM coursework, and better preparing them for calculus. The administrative support was necessary to provide immediate funding for the FLC to create a pilot version of the course, and a longer-range commitment for the human and financial resources needed to provide professional development workshops for faculty interested in joining the FLC. The project also lined up with institutional strategic goals around student success and the valuation of teaching excellence, namely, incentivizing the redesign of gateway courses that "will follow national best practices for effective pedagogical approaches" [14, pg. 20–21].

Step 2: Meet with colleagues from courses directly impacted by Precalculus.

Next, we met with colleagues from several courses: Trigonometry, Precalculus, and Calculus. We outlined a plan to investigate whether the teaching and learning of precalculus through quantitative and covariational reasoning produces relevant conceptual understanding and preparatory algebraic skills acquisition for our FIU students. The proposal was well-received due to the presence of the localized study. The Trigonometry and Precalculus faculty ensured that the content chosen by the FLC lined up with not only the SUS requirements (Figure 1) but also with their individual courses. The Calculus faculty also helped determine which material should take precedence for their downstream courses and which could be deprioritized. Following chair approval, a small pilot of three sections was conducted in Fall 2018, which was then expanded to six sections in Spring 2019.

Step 3: Identify Student Learning Outcomes (SLO).

Fortunately, the curriculum uniquely situated itself to deliver a robust set of learning outcomes. Our goal was to create a set of outcomes that focused on these three main strands:

- Novel conceptual knowledge acquisition, for example, the understanding of rational functions producing a relative size measurement of the dependent values of two functions based on the independent value.
- Development of problem-solving and reasoning abilities, for example, reasoning a pattern of linked change between the time elapsed and the mass of the bacteria quantities (Figure 2).
- Mastery of mathematical processes, for example, applying knowledge of one-to-one functions to solve inverse trigonometric equations.

"Trigonometry – MAC 1114:

1. Trigonometric Functions, their Properties and Graphs 2. Inverse Trigonometric Functions, their Properties and Graphs 3. Trigonometric Identities 4. Conditional Trigonometric Equations 5. Solutions of Triangles 6. Vector Algebra 7. Parametric Equations 8. Polar Coordinates 9. Applications"

"Precalculus Algebra – MAC 1140:

1. Polynomial, Rational, and other Algebraic Functions, their Properties and Graphs 2. Polynomial and Rational Inequalities 3. Exponential and Logarithmic Functions, their Properties and Graphs 4. Piecewise Defined Functions 5. Conic sections 6. Matrices and Determinants 7. Sequences and Series 8. Mathematical induction 9. Binomial Theorem 10. Applications"

"Precalculus Algebra/Trigonometry – MAC 1147:

This course is a one-semester course encompassing the topics of MAC 1140 'Precalculus Algebra' and MAC 1114 'Trigonometry'. MAC 1147 = MAC 1140 + MAC 1114. Please see the topics for MAC 1140 + MAC 1114 for MAC 1147 topics."

Figure 1. Florida Department of Education statewide course descriptions [13].

Additionally, it was important to us to develop a community within the classroom through group work and discussion to better strengthen student involvement and a sense of self-efficacy. Our finalized list of SLOs was:

(1) Recognize the key ideas foundational to Calculus
(2) Reason about two quantities that change in tandem with each other
(3) Demonstrate a process view of functions
(4) Formulate, represent, and solve novel real-life problems
(5) Apply algebraic procedures accurately and efficiently
(6) Explain and justify mathematical assertions
(7) Communicate mathematical concepts through writing and group discourse

Step 4: Create a weekly schedule.

Once the topics were decided upon, we began the creation of the weekly schedule, shifting material back and forth to give space for more content-heavy weeks. During this process, our primary focus was on ensuring coverage of all the necessary material at a manageable pace for students while keeping in mind the overarching student learning outcomes for the semester. Next came the creation of the pre-class content within the online platform. The goal there on the part of the FLC was two-fold: from a practical perspective, to eliminate as much exposition from the class meeting as possible to allow for a more productive group learning environment, and to require that students prepare for class by thinking about new material and doing the beginning development of upcoming concepts independently.

An advantage of the online *Pathways* materials was the focus on student learning through a constructivist process, leveraged by applets engaging students in continuous reasoning. The following proposed set of guiding principles for designing radically constructivist curricula is put forward by Thompson

(1) Be problem-based.
(2) Promote reflective abstraction.
(3) Contain (but not necessarily be limited to) questions that focus on relationships.
(4) Have as its objective a cognitive structure that allows one to think with the structure of the subject matter.
(5) Allow students to generate feedback from which they can judge the efficacy of their methods of thinking [29, pg. 200].

The pre-class applets coupled with in-class discussion and investigation provide students opportunities to construct more extensive meanings. For example, the applet in Figure 2 promotes the development of a notion of the logarithm as denoting the amount of change in the exponent needed to yield a certain ratio of powers. The applet pursuing the relationship for the static whole number increment of $\Delta t = 1$ allows instructors then to simultaneously reference the juxtaposition of the motion of time through a continuum and the variable nature of the amount of change in time, hence supporting students in acquiring a dynamic process view of

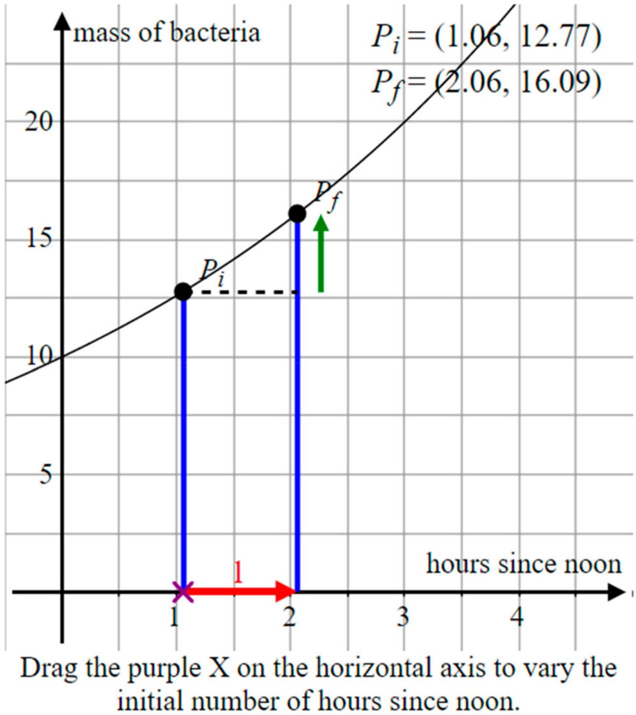


Figure 2. An online manipulative exploring growth factors ([23]). *Reprinted with permission by Rational Reasoning, LLC (Copyright 2019).*

the idea of logarithm:

$$\log_{1.26}(y) = \Delta t \quad \text{where} \quad \frac{1.26^{t+\Delta t}}{1.26^{t}} = y$$

Step 5: Choose the in-class work.

Our next task was to identify in-class questions from the *Pathways* workbook that would best realize the high-level student learning outcomes we outlined for the course. The problems were chosen to not only address new content but also best facilitate group learning. The FLC chose problems that would keep students both engaged with one another as well as motivated, especially for those who had experienced lectures as the main instructional strategy in their past math courses.

Step 6: Create the out-of-class assignments.

Last, we worked together to create both online and offline weekly homework. The online homework was graded automatically and provided both conceptual and computational practice. While promoting computational fluency was not the purpose of choosing this curriculum, given the department's need and value for computational fluency, the *Pathways* developers were asked to write additional computational problems into their existing online homework and the FLC authored original questions as well. The hope was that this approach would balance faculty requests for more computational practice opportunities with the goal of introducing more activities that developed conceptual understanding and mastery. The purpose of the written homework was to focus on student writing: allow faculty to measure students' knowledge, provide feedback when required as well as prepare students for exam problems.

Step 7: Further develop the LA model for class time.

The Mastery Math Program calls for the incorporation of Learning Assistants (LAs) into both the classroom and the Mastery Math Lab, where students are required to spend one hour a week. As affiliated participants of the FLC, the LAs needed additional training to prepare them to work with the new curriculum; they worked on problem sets ahead of time and attended weekly prep sessions (90 min per session) where they participated in group work, presented their ideas and solutions on white boards, and formulated likely student questions. LAs were pressed with the importance of encouraging students to draw diagrams and define variables whenever applicable and ask leading questions to help the students make progress without giving the answer.

Step 8: Establish weekly meeting norms.

It was important for the FLC to establish weekly meetings throughout the semester, where faculty could come together in a collegial environment, as is the norm in the lower-level math courses at FIU. The purpose of the meetings was to create a supportive space where we could discuss any issues from the previous week, review upcoming content, and offer advice on how to best approach material. Faculty shared their evolving conceptions of precalculus through the teaching and reflection on the course. For example, faculty began to think of functions as relationships between the values of two linked quantities rather than a decontextualized x and y. Another key insight being faculty picking apart the quantitative structure of circular motion in context, by starting with two quantities and building a third quantity from them. For example, when modeling the vertical distance above the ground of a rotating Ferris wheel, the amplitude and midline of the sinusoidal function are not simply numerical but are conceived as quantities used in constructing a measure of the vertical distance above the ground. Similarly, the quantitative operation between the constant angular speed and the independent variable time elapsed helps to make sense of the quantitative nature of the argument of the sine function. Previously the scaling and shifting had only arithmetic meaning for faculty.

Additionally, the group would use the meeting time to work on assessments. Test questions were to be written individually, with the aim of faculty further engaging with the material and developing a better understanding of the course. Questions would then be discussed and reflected upon in a communal setting. All test versions were constructed with similar question types and difficulty level to ensure uniformity across all sections of the course.

5. GRADING

5.1. Grading Structure

Course grades were based largely on exam scores (approximately 75%), with other assignments still worth points so that students completed those tasks and grasped the value they added to their learning [30]. Written homework, weighted at 8%, was given to further the individual development of students' problem-solving abilities as well as their written mathematical explanatory skills. For example, in Figure 3, when covering quadratic functions, our aim was to have students explain the contextual meaning of vertex and zeros embedded inside the recently covered content of transformations (SLOs 4 and 7, above). Written homework also provided faculty the opportunity to give detailed feedback to help students better prepare for their upcoming assessments. In general, students were assessed on their setup of problems and conceptual understanding of the material on written homework and assessments.

5.2. Personalized Reassessments

The FLC's viewpoint on assessments evolved following the pilot semester and starting in Spring 2019 assessments were organized to foster learning experiences with the goal of students viewing them as checks on their learning. This was motivated by an interest in ultimately adopting specifications-based grading (SBG), made

An ice cream shop finds that its weekly profit P (measured in dollars) as a function of the price x (measured in dollars) it charges per ice cream cone is given by the function k, defined by $k(x) = -125x^2 + 670x - 125$ where $P = k(x)$.

a) Determine the maximum weekly profit and the price of an ice cream cone that produces that maximum profit.
b) The cost of the ice cream cone is too low then the ice cream shop will not make a profit.
   Determine what the ice cream shop needs to charge in order to break even (make a profit of $0.00).
c) If the cost of the ice cream cone is too high then not enough people will want to buy ice cream. As a result, the weekly profit will be $0.00. Determine what the ice cream shop would have to charge for this to happen (the profit to be $0.00).
d) The profit function for Cold & Creamy (another ice cream shop) is defined by the function g where $g(x) = k(x - 2)$.
   1. Does the function g have at the same maximum value as k?
   2. What is the price per ice cream cone that Cold & Creamy ice cream shop must charge to produce a maximum profit? Explain.
e) Describe the meaning of $h(x) = k(x) - 125$ and then compare/contrast the maximum value for each function.

Figure 3. Example of a written homework problem on quadratic functions ([10, p. 196]). *Reprinted with permission by Marilyn Carlson (Copyright 2020).*

successful at FIU in a physics without calculus course [21, 24]. The *Pathways* curriculum would further benefit from SBG due to its basis in longitudinal concept development [2].

To achieve this goal, we decided to implement personalized retesting. For Spring 2019, retesting was done only on the first three of the four-term tests. It was personalized in the sense that for each individual student, the lowest performing segment of test questions (around 30% of the test's overall questions) was identified. The student then received an opportunity to reassess the learning goals underpinning those questions. The FLC worked together to create a bank of retest questions that were similar to the original test questions but would require students to demonstrate the ability to recognize the material in fresh contexts.

Figure 4 shows an example of an original Test 1 question (left) on modeling dynamic function relationships. Its variant version from Retest 1 (right) assesses the same reasoning abilities while using a new applied context.

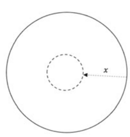
You are given a circular sheet of paper with a radius of 18 inches. You are asked to cut a line inwards and then cut out a smaller circle whose center is the same as the original circle (see picture below). Let x represent the length of the cut, measured in inches.  a. What is the radius of the smaller circle in terms of the length of the cut you made? b. Using function notation write the area of the smaller circle (measured in in^2) in terms of x (measured in inches). c. What is the domain of this function?	Imagine that you have rolled out a piece of dough into a rectangle that measures 12 inches in width by 9 inches in length. You begin to stretch it, adding $x$inches to the length while removing x inches from the width. a. What is the length of the new rectangle, in terms of x? b. What is the width of the new rectangle, in terms of x? c. Use function notation to define the area of the new rectangle (measured in in^2) in terms of x (measured in inches). d. What is the domain of this function?

Figure 4. Differences in a reassessed question.

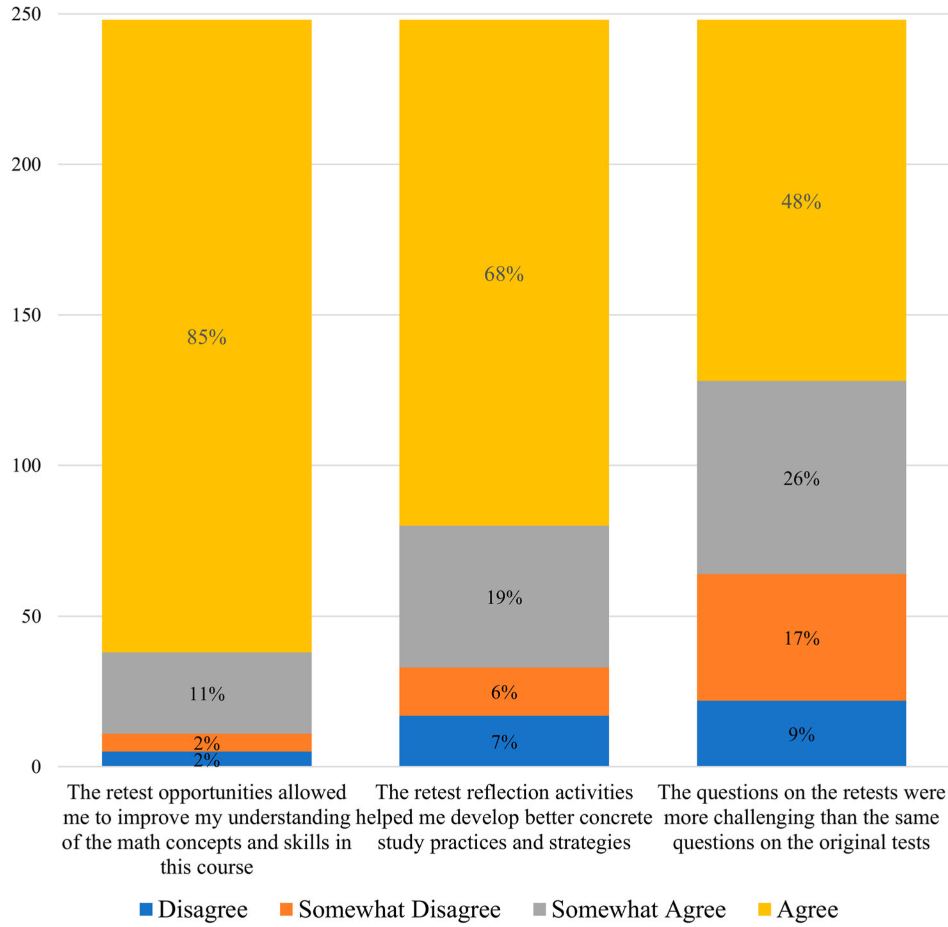


Figure 5. Responses of $N = 248$ *Pathways* student respondents on the Spring 2019 FIU Student Perceptions & Behavior Survey, see [16].

The reassessments were done at scale, and students from all sections tested together in the math labs outside of class time. Students were required to complete an entry ticket for each of the three reassessments: the first one a metacognitive reflection, the second an error analysis assignment, and the last a rate yourself activity. Student perception of the retest opportunity was recorded in the FIU Gateway survey, a campus-wide survey distributed at the end of each semester. Results are shown in Figure 5.

5.3. Cognitive Connection

Additionally, the FLC was interested in whether assessments reliably aligned with the course learning objectives. This included the question of whether there was enough procedural and skills-based knowledge being measured, in addition to evaluating the presence of more ambitious problem-solving and concept development strands. An analysis of tests 1–4 was done using the *Exam Characterization*

Table 1. Some item orientation percentages for the 129 items.

Item orientation	%
Recall and Apply Procedure/Remember	54.26
Understand	13.95
Apply Understanding	31.01

Table 2. Some item representation and item format percentages for the 129 assessment items.

Item Representation	
Applied/Modeling Representation	54.26%
Item Format	
Word Problem Format	52.17%

Framework diagnostic [28]. The four tests were composed of 129 distinct items, and coding with the three-dimensional framework was done by one of the authors. The analysis indicated that more than 45% of the items required a higher cognitive demand than "recall of" and "carry out" a procedure, and over 52% of the items were either applied/modeling representation or in word problem format (Tables 1 and 2).

6. SCALING UP

Effectively scaling up the pilot required more than just handing the work over to incoming faculty, as successful teaching of the *Pathways* curriculum requires a deeper understanding of the philosophy behind the course and a vision of the recurring themes that prevail throughout the semester [19]. Selected faculty would become an essential part of the FLC by offering reflections and suggestions as to how to further improve the course.

The second round of faculty was chosen from those who were already familiar with the mastery process found in our introductory math courses that use the Mastery Math Lab and had attended several professional development opportunities offered to the department. Additionally, these five faculty had ample experience employing high-touch practices in their courses. "High-touch" means that faculty closely follow their students' progress, using various modes of communication to promote more meaningful student–faculty engagement [25]. Most importantly, this particular group was interested in exploring the curriculum and open to change. In order to develop the organic growth of the project, there was a desire to keep the initial group small.

In this way, the FLC was expanded, also including two key Mastery Math Lab personnel, with the goal of meeting every other week for Pathways Prep sessions during the fall semester to prepare for the spring. Fortunately, the idea of faculty collaboration was well known among this particular cohort as it is a normative expectation on the part of any faculty teaching in the Mastery Math Model, which has been in place since 2012.

FLC meetings focused on preparing faculty to support student understanding of ideas addressed in upcoming lessons. Faculty completed problems in the *Pathways* workbook before each session in order to experience student initial exposure to and struggle with the material. This was aligned with our goal of using the FLC as a mechanism to cultivate faculty progression from algorithmic and procedural instruction to teaching centered on student cognition. During the working groups, the faculty discussed these problems in detail, trying to best understand how the content utilizes the philosophy of *Pathways* as well contrast it with their previous teaching of the material. They reflected on questions such as:

(1) What were the major ideas in this investigation?
(2) How have you taught these ideas previously?
(3) What understanding of these ideas are needed to complete this investigation and how do these understandings compare with what is expected in standard curricula?
(4) What elements/questions epitomize the *Pathways* learning philosophy? How?
(5) What are the key points that students should get in this activity? How do you make sure that happens?

At points during the FLC discussion, it was helpful to review the detailed instructor notes and solutions to compare our image of the targeted learning goals with those of the *Pathways* authors. The FLC members discussed the rationale for question scaffolding within the investigation. However, at times we had to remind ourselves to examine the scaffolding from the perspective of its impact on student learning. For example, the idea of function composition is introduced in the context of the pebble dropped in a lake problem (Figure 6). The question prompts students to create a function formula for the area of an expanding circle in terms of the time elapsed since the circle began expanding. The curriculum poses questions to help students visualize the quantities in the problem context prior to asking them to construct the formulas for the area in terms of radius and radius in terms of time. However, during FLC sessions, faculty tended to write the final composite function immediately upon reading the initial description of the problem, indicating that faculty were more focused on getting the answer than considering the thinking that would enable students to construct a meaningful answer. This is one illustration of how the FLC members' discussions and engagement with the curriculum tasks led to their shifting the instructional goals more toward the development of students' thinking in contrast to focusing only on producing the right answer.

The expanded FLC met a total of six times during the fall semester to discuss short papers on pedagogy, learning ideas central to the curriculum, and classroom practices (e.g., in-class quizzing, effective use of learning assistants, timed completion). The focus was not just on working through the problems in the curriculum but also on understanding what thinking would enable students to make sense of and construct a response on their own. Weekly meetings then commenced once the spring semester was underway with faculty discussing issues they had faced the

A pebble is thrown into a lake and the splash creates a circular ripple. The radius length of the circular ripple increases at a constant rate of 7 cm per second. Your goal is to determine the area (in square centimeters) inside the ripple in terms of the number of seconds since the pebble hit the water.

a) Draw a picture of the situation and label the quantities. Imagine how the quantities are changing together. Discuss in your groups what processes need to be carried out to determine the area inside the ripple when the number of seconds since the pebble hit the water is known.
b) What quantities are varying (changing) in the situation and how are they changing together?
   i. As the time since the pebble hit the water increases, how does the radius of the ripple change?
   ii. As the radius of the circular ripple increases, how does the area of the ripple change?
   iii. As the time since the pebble hit the water increases, how does the area of the circular ripple change?
c) Define a function f that determines the radius (in centimeters) of the circular ripple in terms of the number of seconds, t, since the pebble hit the water.
d) Define a function g to determine the area (in square centimeters) of the circular ripple in terms of the circle's radius length, r (in centimeters).
e) Describe the process of evaluating $g(f(4.5))$.
f) In the expression $g(f(4.5))$ describe what quantity and unit of measurement is associated with each of the following.

	Quantity	Unit of measurement
i) 4.5	____________	________________
ii) $f(4.5)$	____________	________________
iii) $g(f(4.5))$	____________	________________

g) Define a function h that determines the area of a circular ripple (measured in square centimeters) as a function of the time t (measured in seconds) since the pebble hit the water.
h) Compute the value of $h(4)$ and the value of $g(f(4))$. What do you observe?

Figure 6. Example of a workbook problem designed to address student misconceptions on chaining together two functions ([10, pp. 91–92]). *Reprinted with permission by Marilyn Carlson (Copyright 2020)*.

previous week, how to better hone the curriculum to our students' needs, what to expect in the coming weeks, and write exams together.

Two more faculty Prep Session Workshops were then run one in the spring to prepare for the summer teaching, and one in the summer to prepare for the fall teaching. These cohorts were different in that they also included tenure-track faculty who do not teach Precalculus. The aim was to include some of them in a consultant role for the FLC, their participation providing insights into the content focus of the *Pathways* curriculum and its approach to preparing students for calculus. In turn, they also became more aware of their students' background and preparedness for calculus.

7. RESULTS

During Fall 2018 and Spring 2019, we conducted a study comparing sections with the *Pathways* curricular treatment and Non-*Pathways* control sections. Below we describe the findings of data that we collected and used for the comparison. First, we examined student outcomes on two different types of assessments, a pre/post inventory, as well as final exam questions. Also, pass rates in the MAC 1147 courses themselves were calculated, with a C or higher considered passing, in addition to downstream pass rates in Calculus I. Finally, we collected open-ended survey responses. In the following subsections, "P" represents students in a *Pathways* section, and "NP" represents students in a section of the study that did not use *Pathways*.

7.1. Pre/Post Assessments

The Calculus Readiness Exam (CRE; [6]) was run pre and post (see Figure 7). This 30-item multiple-choice instrument was selected over the Precalculus Concept

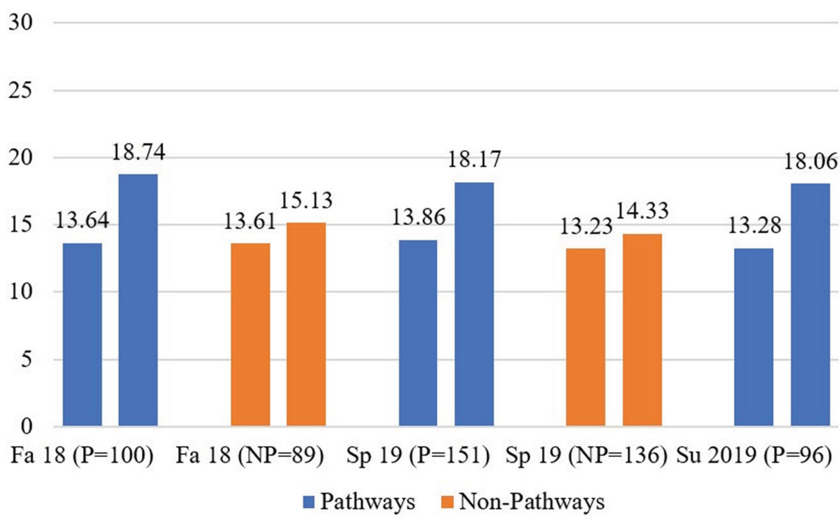


Figure 7. Average number of correct responses (out of 30 total questions) per student on the CRE.

Table 3. Gains on the CRE.

Term	Normalized gain		Raw gain	
	P	NP	P	NP
Fall 2018 (P = 100, NP = 89)	31.17%	9.27%	37.39%	11.17%
Spring 2019 (P = 151, NP = 136)	26.70%	6.56%	31.10%	8.31%
Summer 2019 (P = 96)	28.59%		35.99%	

Assessment (PCA; [9]) because it contained trigonometry content, whereas the PCA contained none. Students were given fifty minutes and were instructed that they would receive one bonus homework point for each question where the work shown was indicative of thoughtful effort (Table 3).

The pre/post matched data showed that while the pre-averages of both *Pathways* and Non-*Pathways* students were only incrementally different, the gains seen by the *Pathways* students were far greater. The data was used to further streamline the class and better align the weekly instruction with the overarching course objectives.

7.2. Common Final Exam Questions

Additionally, a set of ten fixed multiple-choice final exam questions was given during both Fall 2018 and Spring 2019. These questions focused on procedural pre-calculus knowledge. The data in Tables 4 and 5 show *Pathways* performing better on seven of the ten questions during the fall, and five of the ten questions during the spring. The data were used to highlight the areas in which the *Pathways* students were lacking procedural technique skills in order to make improvements for the following year.

7.3. Course Outcomes

Course outcomes from Fall 2018 and Spring 2019 show a higher pass rate in both semesters for the treatment group as well as a lower fail rate, with student populations in both groups close in number. The drop rate for the *Pathways* sections was initially higher but evened out in the spring semester (Figure 8).

Table 4. Data on common final exam questions during Fall 2018.

Final exam question topic	Percent Correct	
	P = 119	NP = 119
Sequence	93%	83%
Ellipse	79%	59%
Quadratic Transformation	95%	91%
Trigonometric Transformation	69%	60%
Rational Function	71%	70%
Logarithmic Equation	34%	25%
Domain	59%	52%
Polynomial Inequality	64%	66%
Difference Quotient	59%	71%
Finding Trigonometric Value	41%	56%

Table 5. Data on common final exam questions during Spring 2019.

	Percent Correct	
Final exam question topic	P = 266	NP = 300
Sequence	88%	87%
Ellipse	59%	43%
Quadratic Transformation	92%	91%
Trigonometric Transformation	67%	64%
Rational Function	58%	51%
Logarithmic Equation	29%	32%
Domain	43%	58%
Polynomial Inequality	60%	70%
Difference Quotient	61%	69%
Finding Trigonometric Value	35%	44%

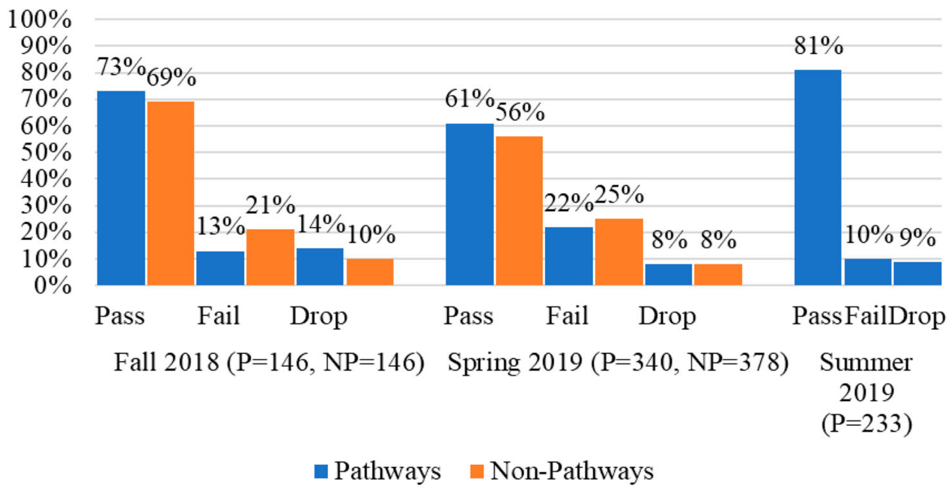


Figure 8. Data on MAC 1147 course outcomes.

It is important to note that there were several uncontrolled variables including the disparate nature of the curricula, the difference in assessment practices as well as the coordinated nature in which the *Pathways* sections were run. The spring faculty cohort consisted of those who were teaching the course a second time, as well as new members of the FLC who had undergone the *Pathways* professional development aimed at increasing their understanding of the learning goals embedded in the *Pathways* curriculum. What is interesting to note is that the drop rates in the *Pathways* sections from Fall 2018 to Spring 2019 significantly decreased, likely due to the implementation of reassessment opportunities. Additionally, experience from the fall semester allowed for a revision and refinement of both the in-class activities as well as messaging to students, helping them to better recognize and value the recurring themes of the curriculum and highlighting how this foundation prepares them for calculus.

7.4. Follow Through Data

The Fall 2018 MAC 1147 students who continued on and took either MAC 2311 (Calculus I) or MAC 2281 (Calculus I for Engineering) during Spring 2019 were

Table 6. Data on longitudinal Calculus course outcomes.

Pass thru pass rates	P	NP
MAC 2311 MPC (P = 15, NP = 13)	93%	85%
MAC 2311 Overall (P = 63, NP = 108)	76%	75%
MAC 2281 (P = 10, NP = 23)	80%	78%
Calculus I (P = 73, NP = 131)	77%	76%

tracked and had the pass rates listed in Table 6. The MAC 2311 Modeling Practices in Calculus (MPC) Curriculum subgroup denotes reformed sections employing modeling practices [11]. The MPC curriculum is centered on a reconceptualization of the classroom learning environment, including active learning, culturally responsive teaching, and patterning and replicating the practices of mathematicians in the learning of concepts in calculus.

The data show that students who were enrolled in the *Pathways* sections in Fall 2018 had on average slightly higher pass rates than those who were in the Non-*Pathways* sections. This was true across all forms of calculus taught at the university in the spring. Unfortunately, the population sizes were often small.

In Spring 2019, the PCA was given at the beginning of MAC 2311. We performed Welch's t-test to compare the mean GPU and PCA score for students entering MAC 2311 from each type of Precalculus. The results below summarize this analysis of end-of-course Precalculus grades and demonstrated knowledge on an unfamiliar assessment one month later (Tables 7 and 8). Only two of the twenty-five PCA items are on the CRE, so we were not concerned about students having just seen the questions at the end of the previous term.

The Fall 2018 MAC 1147 passers who took this assessment, when organized into P and NP, did not have a statistically significant difference in average GPU ($t = 0.29$, $p = 0.771$). However, there was a statistically significant difference ($t = 2.69$, $p = 0.009$, d $= 0.56$) on PCA scores. This outcome supports the claim that *Pathways* students performed better on the following semester's pre-test. Since higher scores on the PCA have been shown to lead to higher levels of success in Calculus [9], we conclude that students in the *Pathways* sections were better prepared to be successful in Calculus.

Table 7. Comparison on Grade Point Unit (GPU; A = 4.0, A- = 3.67, B+ = 3.33, B = 3.0, B- = 2.67, C+ = 2.33, and C = 2.0).

	GPU Mean	Sample standard deviation	*t*-value	*p*-value
Pathways ($N = 35$)	3.105	0.7	0.29	0.771
Non-Pathways ($N = 60$)	3.061	0.7		

Table 8. Comparison on PCA performance.

	PCA mean	Sample standard deviation	*t*-value	*p*-value	Cohen's *d*
Pathways ($N = 35$)	12.1	3.1	2.69	0.009	0.56
Non-Pathways ($N = 60$)	10.3	3.5			

Table 9. Student perspectives on feedback from [16].

#	Question: How much helpful feedback did you get on your work for this class?	Non-Pathways		Pathways	
4	I got enough helpful feedback for my needs.	30.57%	70	41.27%	104
3	I got some helpful feedback but not as much as I needed.	25.33%	58	34.52%	87
2	I got some feedback but it was not that helpful.	17.47%	40	15.87%	40
1	I got little or no feedback.	26.64%	61	8.33%	21
	Total	Total	229	Total	252

Table 10. Student perspectives on classroom engagement from [16].

#	Question: Which best describes your math class?	Non-pathways		Pathways	
1	The instructor lectures with no/few interactive activities	51.53%	118	4.78%	12
2	Mostly lecture, but there is usually some interactive activity	31.88%	73	6.77%	17
3	About half lecture and half activities	12.23%	28	32.67%	82
4	Mostly interactive activities, with little/no lecture	4.37%	10	55.78%	140
	Total	Total	229	Total	251

7.5. Student Perceptions

At the end of the spring semester, the FLC administered a survey to get a better understanding of how students were feeling about the course and whether the scaled-up expansion had instructors implementing the active-learning design of the course with high fidelity (Tables 9 and 10).

Our findings show that the majority of the *Pathways* sections were being run in a student-centered environment. Additionally, while most of our students were receiving meaningful feedback, our data suggested that students desired more detailed feedback on their mathematical thinking.

7.6. Conclusions

The above data together show that *Pathways* was more effective than the traditional curricular treatment in preparing students for unreformed calculus, as well as for a version of reformed calculus. Students benefited from and appreciated a reassessment process that fosters learning beyond an initial exam. In addition, the success of the personalized reassessments justified funding the next phase of the *Pathways* adoption, creating a full-scale SBG version of the course.

8. REFLECTIONS

There are several takeaways from the first year of implementation. First is the importance of creating a Faculty Learning Community within which to house the project. Implementing a new curriculum that is radically different from previous iterations of the course requires faculty members who are not only dedicated to successful transformation of a class but are also interested in debating and discussing current educational research to create work that is both student-centered as well as rich in content. Additionally, starting with a smaller FLC and then expanding its membership allows for a fresh perspective during the process to help move the work forward and encourage reflection on the part of the original faculty participants.

Second is the necessity of providing professional development workshops to properly prepare faculty to teach the curriculum. It can take time and a deep immersion in the content for faculty to both shift away from their traditional teaching of Precalculus as well as develop an understanding of covariational reasoning, one of the central themes of *Pathways*. That, along with having an integrated view of the curriculum is required to create a coherent picture of the course for students. Being a part of an FLC and attending workshops in addition to weekly semester meetings for faculty currently teaching the course provided support for faculty and helped them understand that teaching is not just about the performance in the classroom but also about changing student thinking along the way.

Next is the role of messaging on the part of the faculty. Those who were not fully bought into the new curriculum often transmitted that uncertainty to their students who in turn found fault in the methodology of the course. Messaging should also include reinforcing the importance of student engagement with the material, as active learning and quantitative reasoning are new ways of learning and thinking about mathematics for the majority of them. Additionally, framing reassessment opportunities as just a continuation of the learning process as opposed to punishment for those who have not yet mastered the material helps avoid a deficit-based approach to the course. In particular, the FLC is instrumental in developing and adapting the messaging over the course of a semester as a result of classroom experience. For example, faculty who had taught the class in the fall were able to help new instructors craft messaging specific to the active-learning nature of the class in order to help students better appreciate the purpose of certain instructional strategies.

Likewise, the restructuring of the course is an iterative and often non-linear process. In our first year, we found that further refinement of the in-class group work was necessary to provide more focus on the essential investigations within the curriculum. Also, while it is not always easy to balance a traditional demand for procedural fluency with an attempt to build student conceptual understanding, a deeper exploration of course materials helped us see that procedural fluency is being addressed through the process of building student conceptual understanding. We were able to remove or adjust several of the extra worksheets we had built simply by choosing other problems to work on during the class time.

There is a final and perhaps most overarching consequence that we offer to any institution or department with interest in piloting and scaling a research-based curriculum: early on, engage a set of faculty and other partners, whether broad or narrow, in a collaborative process of sensemaking. Conversations should center around the relevant factors and elements of the course inside the institutional ecosystem alongside a reformation of the student experience.

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