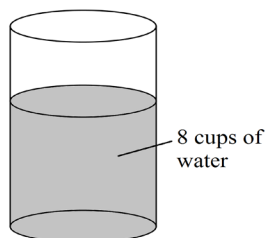


We have practiced visualizing and representing the varying value of quantities using variables and expressions. In this investigation we will use **number lines** to represent and determine a quantity's value.

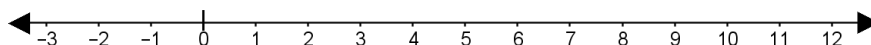
- The value 0 on a number line represents the starting point from which a quantity is being measured.
- Values to the right of 0 are positive and values to the left of 0 are negative.
- In a **uniform scale** equal changes in value are represented with equal lengths on the line.

1. Imagine a cylindrical vase that holds 12 cups of water. The vase now has 8 cups of water as shown in the illustration to the left and below.

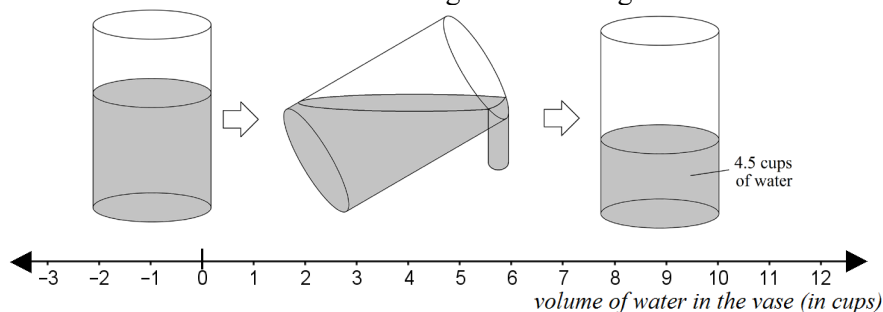


Every “1 unit” length on the number line to the right of 0 represents 1 cup of water in the vase.

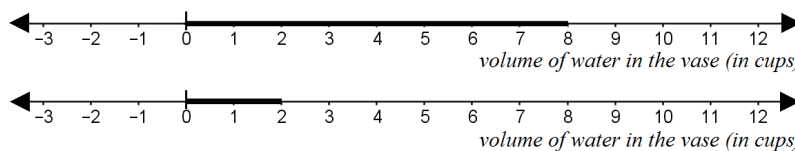
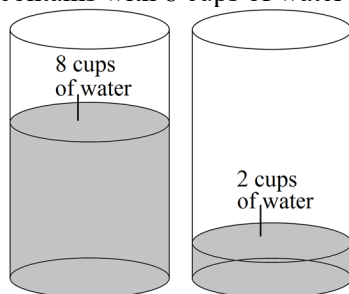
- a. On the number line, represent 8 cups of water in the vase by constructing a line segment 8 units to the right of 0.



- b. Imagine pouring some water out of the vase and the vase now has 4.5 cups of water. Represent this amount of water in the vase with a line segment to the right of 0.

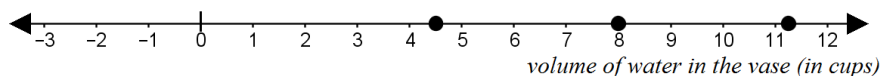


2. By carefully drawing a uniform scale, we can visually compare different values by comparing the corresponding lengths along the number line. For example, imagine two identical vases, except one contains with 8 cups of water and the other with 2 cups of water.

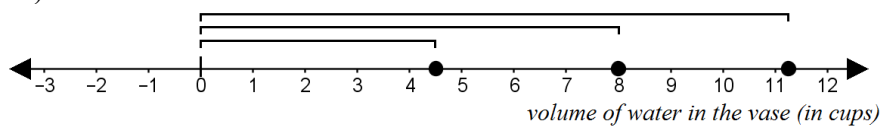


- a. What length represents how many more cups of water are in the vase with 8 cups as compared to the vase with 2 cups?

3. a. Suppose we want to represent 3 volumes all on the same number line (4.5 cups, 8 cups, and 11.25 cups). Is it possible to represent each point with a different line segment on the same number line? Explain.
- b. A mathematical convention is to use points instead of line segments to represent different values of the same quantity. What do the 3 points on the below number line represent?



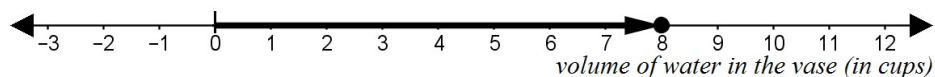
- c. Going forward we will use the mathematical convention of representing an amount of some quantity by simply plotting a point. We should think of this point as a measure of that quantity away from a starting point, labeled 0. What does the point 0 represent on the above number line?
4. It's important to remember that, even though we are only using a point to represent the value of the quantity, that this point represents the endpoint of a line segment starting at 0 and ending at the value (in this case 8).



- a. What does the point 4.5 represent on the line segment?
- b. What does the point 11.25 represent on the line segment?
- c. Use the magnitudes represented by 4.5 and 11 on the number line to represent $11.25 - 4.5$. What else might you characterize this difference?
5. If we want to represent a change in the value of a quantity (instead of just the current value of the quantity) we use a line segment with an arrow on one end (also called a **vector**).

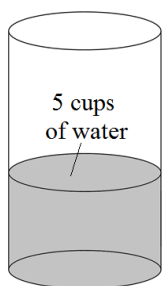
The point where a vector starts represents the starting value, the line represents the change (in the amount of water being added as it passes through all values represented on the line), and the point where the vector ends represents the ending value.

- a. This line segment represents a change in the amount of water in the vase from an initial amount of _____ cups to a final amount of _____ cups.

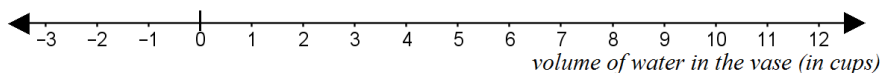


- b. T or F: At some point while water was being added, the vase contained 3.96 cups of water. Explain.

6. Suppose we have a vase containing 5 cups of water.

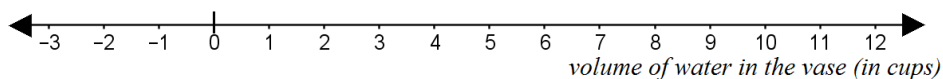


- a. Represent this on the number line.

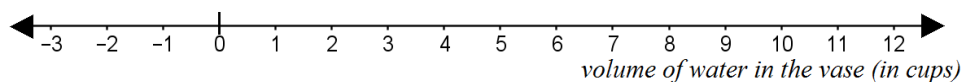


- b. Suppose we then pour in an additional 3.5 cups of water. Represent this change and the final amount of water on the same number line.

7. Suppose the vase has 2.5 cups of water. We then pour in an additional 5.2 cups. Represent the initial amount, change, and final amount of water in the vase on the number line.



8. Repeat Exercise #1 if we begin by pouring 6.1 cups of water into the vase, and then pour in an additional 2.75 cups.



We first represented the initial volume of water in the vase and then we represented a change in the volume of water, to get an ending amount of water in the vase. More generally we can say:

initial amount + some change in the amount
└──────────────────┘
 represents the ending amount

9. Suppose that when you woke up one morning the temperature outside was 57°F and by noon the temperature had increased by 18°F . (Label your number line before beginning.)
 a. Represent this situation using a number line.

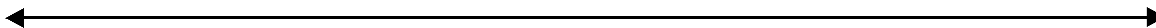


- b. What was the temperature at noon? How is the noon temperature represented on the number line?

10. Repeat Exercise #9 if instead the temperature was -4°F when you woke up and by noon had increased by 13°F .



11. Suppose that when you went to bed one night the temperature outside was -7°F and by the time you woke up the next day the temperature had decreased by 9°F .
- Represent this situation using a number line.



- What was the temperature when you woke up?

In the previous exercises we noticed that sometimes we represented a change by adding and sometimes we subtracted based on whether the value of the quantity increased or decreased. However, there is a way to think about changes so that the general statement

$\text{initial amount} + \text{some change in the amount}$

 represents the ending amount

applies to both increases and decreases.

Changes in a Quantity's Value

From now on we will use *changes in a quantity's value* (or just *changes*) as a general term to refer to both increases and decreases. Increases in a quantity's value will be represented by positive changes and decreases in a quantity's value will be represented by negative changes.

Example:

- If the value of a quantity increases by 4, we will say "the value changes by 4"
- If the value of a quantity decreases by 4, we will say "the value changes by -4 "

Using this method, we can say that the ending amount can always be determined by adding the change to the initial amount. If the initial amount is 13, for example, then

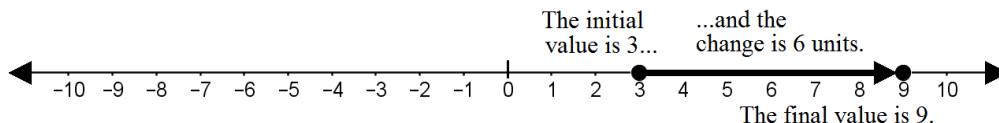
- Change by 4: ending amount is $13 + 4$, or 17.
- Change by -4 : ending amount is $13 + (-4)$, or 9.

For Exercises #9-18, do the following.

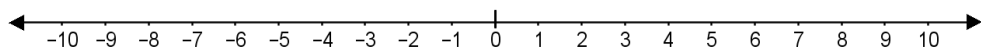
- Represent the sum using vectors on a number line and then give its value.
- Write out in words what the sum represents using the idea of "changes".

The first exercise is completed for you.

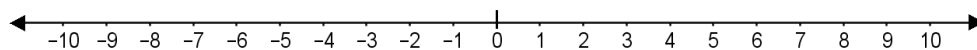
9. $3 + 6$



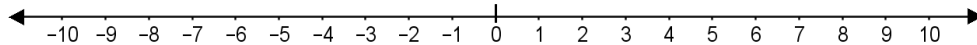
10. $-3 + 7$



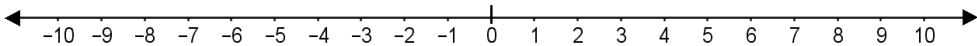
11. $4 + (-5)$



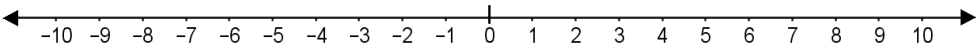
12. $-6 + (-3)$



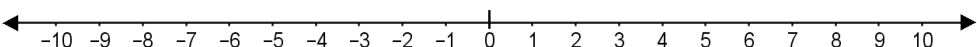
13. $9 + (-3.5)$



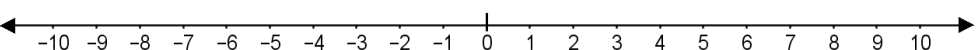
14. $-2.5 + 10.5$



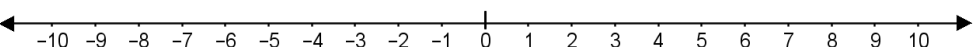
15. $4.7 + 2.4$



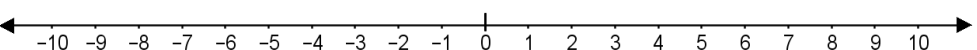
16. $1.5 + (-7.2)$



17. $-3.9 + (-3.2)$



18. $8.6 + (-5.9)$



We end this investigation with a brief comment. In mathematics the idea of *change* is very common so a symbol was chosen to represent the idea. You will often see the symbol *delta* (written “ Δ ”) used to replace the phrase “change in”.