A ***sequence*** in mathematics is an ordered set of objects (usually numbers), such as 2, 4, 6, 8, 10 or 7, 5, 6, 4, 5, 3. The objects in the sequence are called ***terms***.

Finite vs. Infinite Sequences

A ***finite sequence*** is a sequence with a set number of terms. Finite sequences are written like 2, 4, 6, 8, 10 or 2, 4, 6, …, 32.

An ***infinite sequence*** is a sequence with infinitely many terms where the pattern generating the sequence is repeated without end. Infinite sequences are written like 5, 10, 15, ...

1. Consider the sequence $1, 2, 4, 8, 16, \ldots$
   a. Describe the pattern.

   b. Find the next three terms.

   c. Is it difficult to find the value of the 1000$^{\text{th}}$ term? If so, explain why. If not, find its value.

2. Consider the sequence $3, 6, 11, 18, 27, \ldots$
   a. Describe the pattern.

   b. Find the next three terms.

   c. Is it difficult to find the value of the 2,500$^{\text{th}}$ term? If so, explain why. If not, find its value.

3. Consider the sequence $\frac{2}{1}, \frac{5}{4}, \frac{10}{9}, \frac{17}{16}, \frac{26}{25}, \ldots$
   a. Describe the pattern.

   b. Will any of the following be terms in this sequence? If so, which one(s)?

   $\frac{101}{100}$ $\quad$ $\frac{116}{115}$ $\quad$ $\frac{161}{160}$ $\quad$ $\frac{226}{225}$

   c. Is it difficult to find the value of the 50$^{\text{th}}$ term? If so, explain why. If not, find its value.

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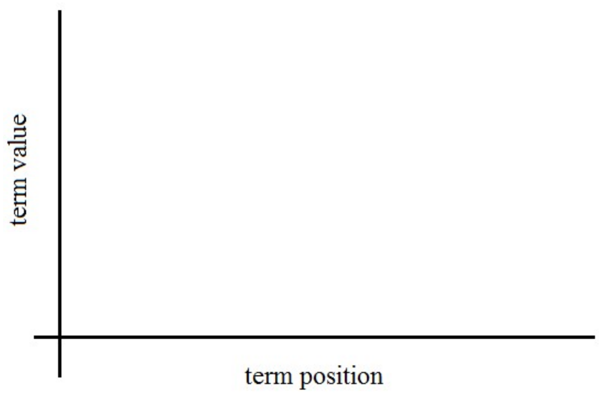
4. Sequences can be thought of as functions in which the input quantity is the ***term position*** [consisting of natural numbers (positive integers)] and the output quantity is the ***term value*** [consisting of the terms of the sequence]. Let's take another look at the sequence in Exercise #3.

input: term position	output: term value
1	2/1
2	5/4
3	10/9
4	17/16
5	26/25

Like other functions, we can create the graph of a sequence. By convention we track the term position on the horizontal axis and the term value on the vertical axis.

a. Before graphing, consider the following question. When you plot the sequence, should the points be connected? Explain.

b. Graph the sequence. Use at least the first six terms.



c. What does the graph suggest about the term values as the term position increases?

<u>The Limit of a Sequence</u>

The behavior you noted in Exercise #4 means that this sequence has a ***limit***, or a constant value that the term values approach as the term position increases.[1]

Note: Only infinite sequences (sequences with infinitely many terms) are said to have limits. Finite sequences (sequences with a definitive end) do not have a limit.[2]

In Exercise #4, the <u>*limit is 1*</u>.

[1] The formal definition (paraphrased) requires that, for a limit to exist, the term values must be able to get as close as we want to the limiting value as the term position increases for the limit to exist. In other words, if we say that a sequence *has a limit of 4*, it means that at some point the term values must be within (for example) 0.1 of 4, and within 0.003 of 4, and within 0.00000007 of 4, etc. for all remaining terms.

[2] Even if the term values of a finite sequence are getting closer to a constant value, eventually the sequence stops and the last term's value is the closest we can get to this constant. Therefore, the term values can't get arbitrarily close to the constant value. For example, consider the sequence 7.9, 7.99, 7.999, 7.9999. The term values are certainly getting closer to 8, but the final term is 0.0001 away from 8. We could never get within 0.000001 of 8, for example, or within 0.00000003 of 8. Therefore "8" doesn't fit the requirement to be a limit of the sequence.

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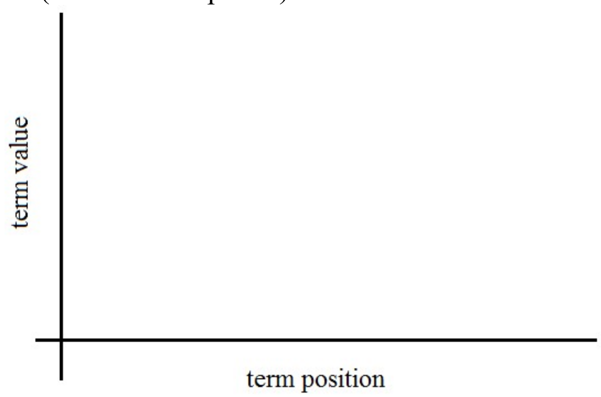
d. We said that sequences can be thought of as functions. Explain to a partner your understanding of this statement. Then explain how they are different from some of the other functions we have studied.

5. A new sequence is generated by taking the difference of 3 and the term values from the sequence in Exercise #4. For example, the first term of the new sequence is $3-\frac{2}{1}=1$. The second term of the sequence is $3-\frac{5}{4}=\frac{7}{4}$.
   a. Graph this new sequence (use at least 6 points).



   b. Does the sequence appear to have a limit? If so, what is the limit?

   c. What is the relationship between the limit of this sequence and the limit of the sequence in Exercise #4? (*That is, is there a mathematical reason why the sequence has term values approaching this limit while the term values of the other sequence approach 1?*)

For Exercises #6-9, do the following.
a) Examine the pattern and then write the next three terms.
b) Graph the sequence (use at least six points).
c) Does the sequence appear to have a limit? If it does, state the limit.

6. 7, 9, 11, 13, ...

7. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...$

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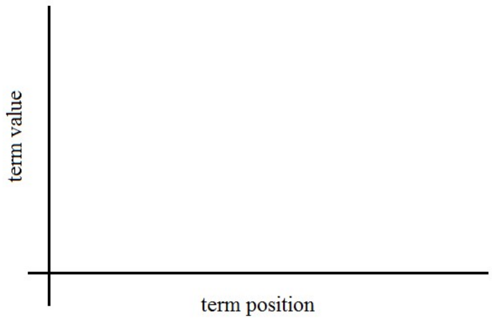
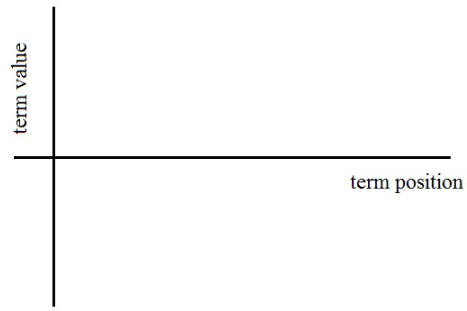
8. $5.1, 4.9, 5.01, 4.99, 5.001, 4.999, \ldots$



9. $1, -1, 2, -2, 4, -4, 8, -8, \ldots$



10. A sequence is formed by choosing any real number x to be the first term, then generating the sequence by taking each term value and dividing by 10 to get the next term value.
    a. Choose a few possible values of x and explore the kinds of sequences produced by following this pattern.

    b. Do all sequences formed in this manner have a limit? If so, what is the limit? Why does this happen?

    c. What if the pattern was instead formed by multiplying the term value by 10 to get the subsequent term value? Would sequences formed in this manner have a limit? Explain.

11. Explain, in your own words, what it means for a sequence to have a limit.

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